THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

On Spectral Estimation and Bistatic Clutter Suppression in Radar Systems

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Abstract

Target detection serve as one of the primary objectives in a radar system. From observations, contaminated by receiver thermal noise and interference, the processor needs to determine between target absence or target presence in the current measurements. To enable target detection, the observations are filtered by a series of signal processing algorithms. The algorithms aim to extract information used in subsequent calculations from the observations. In this thesis and the appended papers, we investigate two techniques used for radar signal processing; spectral estimation and space-time adaptive processing.

In this thesis, spectral estimation is considered for signals that can be well represented by a parametric model. The considered problem aims to estimate frequency components and their corresponding amplitudes and damping factors from noisy measurements. In a radar system, the problem of gridless angle-Doppler-range estimation can be formulated in this way. The main contribution of our work includes an investigation of the connection between constraints on rank and matrix structure with the accuracy of the estimates.

Space-time adaptive processing is a technique used to mitigate the influence of interference and receiver thermal noise in airborne radar systems. To obtain a proper mitigation, an accurate estimate of the space-time covariance matrix in the currently investigated cell under test is required. Such an estimate is based on secondary data from adjacent range bins to the cell under test. In this work, we consider airborne bistatic radar systems. Such systems obtains non-stationary secondary data due to geometry-induced range variations in the angle-Doppler domain. Thus, the secondary data will not follow the same distribution as the observed snapshot in the cell under test. In this work, we present a method which estimates the space-time covariance matrix based upon a parametric model of the current radar scenario. The parameters defining the scenario are derived as a maximum likelihood estimate using the available secondary data. If used in a detector, this approach approximately corresponds to a generalized likelihood ratio test, as unknowns are replaced with their maximum likelihood estimates based on secondary data.

Keywords: Radar Signal Processing, Parametric Spectral Estimation, Space-Time Adaptive Processing, Maximum Likelihood Estimation.

List of Publications

This thesis is based on the following publications:

[A] **Jacob Klintberg**, Tomas McKelvey, “An Improved Method for Parametric Spectral Estimation”. Published in IEEE International Conference on Acoustics, Speech and Signal Processing, May. 2019.

[B] **Jacob Klintberg**, Tomas McKelvey, Patrik Dammert, “Mitigation of Ground Clutter in Airborne Bistatic Radar Systems”. Published in IEEE Sensor Array and Multichannel Signal Processing Workshop.

[C] **Jacob Klintberg**, Tomas McKelvey, Patrik Dammert, “A Parametric Approach to Space-Time Adaptive Processing in Bistatic Radar Systems”. Submitted for publication in IEEE Transactions on Aerospace and Electronic Systems, Dec 2020..

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Finally, and most importantly, I would like to express my gratitude to my friends and family. Särskilt vill jag tacka mina föräldrar för att ni alltid tror på mig, stöttar mig, och har gjort mig till den människan jag är. *(English: Especially, to my parents for always supporting me, believing in me and for raising me to the person I am today.)*

Jacob Klintberg
Gothenburg, March 2021.

Acronyms

RADAR:	RAdio Detection And Ranging
STAP:	Space-Time Adaptive Processing
CFAR:	Constant False Alarm Rate
SINR:	Signal to Interference and Noise Ratio
PDF:	Probability Density Function
DTF:	Discrete Fourier Transform
CUT:	Cell Under Test
IID:	Independent and Identical Distributed
MLE:	Maximum Likelihood Estimate
SCM:	Sample Covariance Matrix
DD:	Direction Doppler
A^2DC:	Adaptive Angle-Doppler Compensation
RD:	Reduced Dimension
DoF:	Degrees of Freedom
LRT:	Likelihood Ratio Test
GLRT:	Generalized Likelihood Ratio Test

Contents

Part I

Overview

CHAPTER 1

Introduction

In 1904 the first radar was patented by Christian Hülsmeyer. It was a pulsed radar, radiating signals generated from a spark gap. Hülsmeyer's radar was based on ideas from experiments done by Heinrich Hertz in 1888, when he discovered the polarization dependent reflection of electromagnetic waves [1]. Although the first patent was registered in the early 20th century, development of the radar first took off during the Second World War. Then, many countries saw the potential to use the radar as a vital tool for detection and tracking of enemy aircrafts.

In modern day society, radar systems are used for more applications than the military. Its ability to work in all weather conditions, as well as for both long and short ranges, makes it to a useful tool in many fields. Thus, radar systems can be found in air traffic control, for weather forecasting and in remote sensing. Moreover, the radar is an important sensor which enables ships to navigate, and provides necessary information for advanced driver assistance systems and autonomous driving in vehicles. The distribution of the radar market per application is shown in Fig 1.1 together with a market forecast [2]. Most of the growth in the market is predicted to come from the automotive industry, strongly connected to the introduction of more autonomous features

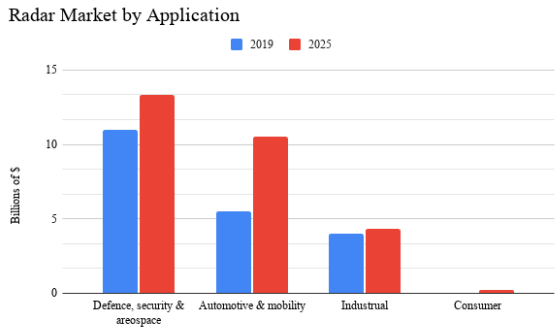

Figure 1.1: Radar market per field of application, and a forecast of the radar market in 2025 [2].

in vehicles [2].

Radar systems is defined as the art of providing targets detections. That is, from electromagnetic waves echoed from the radar surroundings, determine the presence or the absence of interesting targets. To enable this, a radar system is equipped with supporting hardware and software. Various requirements are set both on the hardware and the software depending on the application for the radar to provide detections best suited for the field. Waveforms and directivity of antennas are examples of design choices that may differ between applications. In this thesis we consider military radar applications. In such applications, radar technical dominance has been proven to provide crucial tactical advantages in the field of battle [3]. Consequently, the transmitter antennas capacity to emit electromagnetic energy into space may affect the ability of the radar to detect targets. Moreover, algorithms which mitigate disturbance signals, and usage of various waveforms may infuence the capacity to detect targets at long ranges and with high accuracy. In this thesis and the appended papers, we will further investigate radar signal processing algorithms.

Radar signal processing is a collective of various algorithms which aims to extract the necessary information from radar observations to provide target detections. This implies that the algorithms need to function for various radar scenarios and for many different types of radar configurations. To obtain this flexible behavior, the radar observations are filtered by a chain of signal pro-

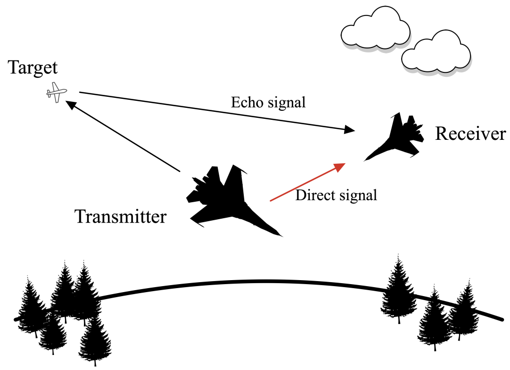

Figure 1.2: Visualization of an airborne bistatic radar configuration.

cessing algorithms. Each algorithm will extract a small piece of information from the observations that can be used in subsequent calculations. Moreover, the radar signals received will be a signature of a complex environment. Thus, the signals will be comprised of many scatters, including scattering from possible targets to be detected and thermal noise from the receiver. To systematically handle this complex behavior, radar signals are often modeled as random signals with certain structure described by a parametric probability density function. Often several parameters describing the radar surrounding are unknown a priori to the processor. Consequently, the signal processing algorithms must adaptively estimate these unknowns for the system to manage a broad spectra of radar environments.

Although adaptive radar signal processing algorithms work for various scenarios and for different radar systems, knowledge about the scenario and the system is necessary when designing the algorithms. That because the behavior of the radar observations will depend on both the scenario and the system. To illustrate this, consider radar observations from an airborne *monostatic* radar system, and an airborne *bistatic* radar system. A monostatic radar system uses the same antenna to both transmit and receive the electromagnetic signals. This is the most common configuration for radar systems. In a bistatic radar system, two antennas that are separated at a large distance in space cooperate with each other to create a functional radar system. Thus, one antenna only emit electromagnetic waves, and the other antenna only receive the echoed signals. A visualization of an airborne bistatic configuration is shown in Figure 1.2. The bistatic configuration has some advantages over the monostatic configuration and has therefore been subject to an increased research interest in recent years. The increased interest arises from a higher degree of digitalization in the radar system, e.g. each antenna in an array is sampled as a separate digital channel, has made it feasible to fully take advantage of the benefits from the bistatic radar configurations. A few of the advantages are the possibility for the receiver platform to operate silently from an electromagnetic point of view. Thus, it will not reveal its position by emitting a signal. Moreover, in a bistatic radar, the transmitter antenna can emit the electromagnetic signals constantly and hence increase the total emitted energy. This since it does not need to shift between emitting and receiving signals, as a corresponding monostatic system operates.

Although the existing advantages for bistatic radar systems, the configuration require additional functionality of the signal processing algorithms. This includes the suppression of the direct signal between the transmitter antenna and the receiver antenna. The direct signal may be of significantly larger power compared to the echoes, and can therefore be of great concern in radar signal processing algorithms, as well as require a high dynamic range of the analog-to-digital converters. Moreover, the processor in the receiver need information about the currently transmitted waveform to properly filter the received echoes. As waveforms typically are changed over time this information must constantly be updated. Furthermore, the behavior of the received echos will be dependent on the position and velocity of the transmitting platform. Also, as the receiver and the transmitter are geographically separated, the dependencies will differ depending on the bistatic range of the signals. Consequently, a geometry-induced range dependent behavior is present in the bistatic radar systems. This phenomenon requires additional processing when mitigating the influence of interference and thermal noise in airborne bistatic systems. In summary, additional processing is required for bistatic radar configurations compared to corresponding monostatic configurations to be able to fully harvest the advantages of the bistatic radar system.

1.1 Thesis contribution

The main contributions of this thesis are the following:

- In Paper A three different methods for parametric spectral estimation are investigated. The considered problem formulation concern estimation of frequency components, and their corresponding amplitudes and damping factors from noisy measurements. In a radar, this problem formulation corresponds to the gridless range-Doppler-angle estimation problem. The main contribution of the paper corresponds to an investigation of how rank management and matrix structure affect the accuracy of the frequency component estimates.

- To mitigate the influence of interference and thermal noise via a space-time adaptive processing technique, an accurate estimate of the space-time covariance matrix is required. In a bistatic radar system, such estimate is non-trivial as the secondary data used for the estimate is non-stationary. In Paper B and C, a method is presented which estimates the space-time covariance matrix based upon the current radar scenario. As the parameters defining the scenario is unknown, the presented method finds their maximum likelihood estimates using the available observations, and calculates the covariance matrix via a model describing the scenario.

1.2 Outline of the thesis

This thesis is divided into two main parts. Part I serves as a general introduction to the subject of radars and to the methods used in the appended papers. In Chapter 2, the fundamentals and working principles of a radar system is introduced. Chapter 3 briefly introduce parametric spectral estimation, and radar detectors are presented in Chapter 4. Space-time adaptive processing is presented in Chapter 5. A summary of the appended papers is presented in Chapter 6. Finally, the thesis is concluded in Chapter 7. In Part II, the three publications are appended.

CHAPTER 2

Fundamentals of Radar Systems

This chapter provides a short summary of the fundamental theory of a radar. For a more comprehensive overview, see e.g. [4].

2.1 Radar signal modelling

A radar system consists of one antenna which emits electromagnetic energy of some waveform into space, and one antenna that receives the reflections of the transmitted signals from objects in the environment. To extract the information from the received signals, the signals needs to be processed. In this section, relations of a monostatic radar is presented. However, the same principles also holds for other configurations.

The distance, or range, to an object is measured by the time the emitted signal takes to travel to the object and back to the receiver antenna. As electromagnetic waves travel at the speed of light, the range R of a monostatic system is

$$R = \frac{c\Delta t}{2} \tag{2.1}$$

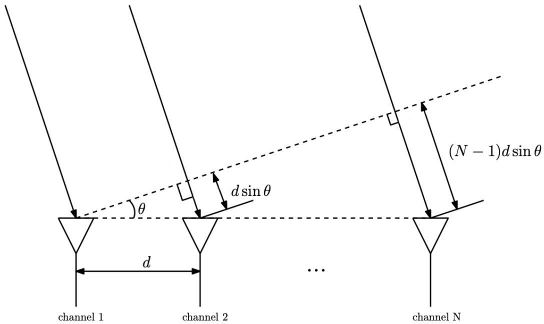

Figure 2.1: Illustration of direction of arrival for an array antenna.

where c is the speed of light and Δt is the duration in time the pulse travels back and forth between the platform and the object [4].

Signals that are reflected by a moving object will be compressed or stretched out in time at the arrival to the receiver. For a narrow band signal this generates a frequency shift compared to reflections of non-moving objects. The shift in frequency is caused by the Doppler effect. Thus, the Doppler frequency is:

$$f_d = \frac{2v_R}{\lambda} \tag{2.2}$$

where f_d is the Doppler frequency, λ is the wavelength the emitted signal and v_R is the relative velocity between the receiver platform and the reflecting object [4].

Signal echoes that originates from long ranges compared to the receiver antenna size will approach the antenna as a parallel wave [4]. For narrow band signals an array antenna will observe a linear phase shift of the received signal between adjacent antenna channels. The phase shift is proportional to the sine of the angle of arrival which the echoes approached the antenna. This phenomenon is illustrated in Fig 2.1. In the figure, a uniform linear array (ULA) is shown with N array channels and distance d between two adjacent array channels. The angle-of-arrival of the signal to the array is θ.

The signal power that returns to the receiver is given by the radar range

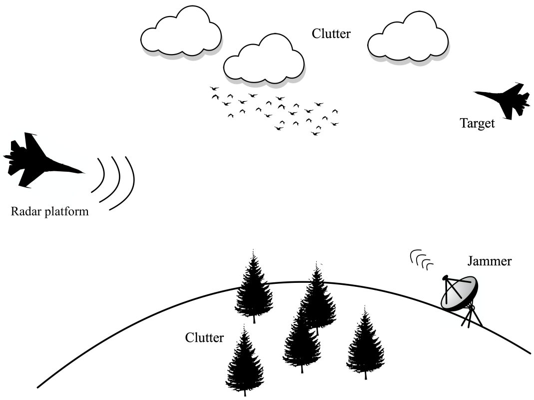

Figure 2.2: Illustration of a radar environment consisting of clutter interference, jamming interference and a target.

equation:

$$P = \frac{\lambda^2 P_T \sigma_{RCS}}{(4\pi)^3} \frac{G_T}{R_T^2} \frac{G_R}{R_R^2} \tag{2.3}$$

where P_T is the power of transmitted signal, R_T and R_R is the range from the transmitter respectively the receiver to the object. The radar cross section of the reflecting object is σ_{RCS}. Gain patterns for the transmitting and receiving antennas are denoted with G_T and G_R, respectively. The angular dependancies of G_T and G_R depends on the type of antenna used, and its configurations [4].

2.2 Radar Operation Environments

In Fig 2.2 an environment of which an airborne radar system operates in is illustrated. Consequently, the observations consists of clutter interference, jamming interference, receiver thermal noise and possible targets. Clutter and jammers are introduced in the subsequent sections.

To be more specific, we now consider a pulsed radar system where the transmitter emits M coherent pulses and the receiver has an array antenna with N distinct channels. The pulses are sampled in the receiver, and sorted in

K range bins depending on the time-of-arrival to the receiver. The range resolution of the sampled signal may be improved using a pulse compression scheme [5]. Denote the radar snapshot at range bin k as $\mathbf{x}_k \in \mathbb{C}^{NM\times 1}$. Assume that the snapshot additively is comprised of clutter interference, $\mathbf{x}_{k,c} \sim \mathcal{CN}(0, \mathbf{R}_{k,c}) \in \mathbb{C}^{NM\times 1}$, jamming interference, $\mathbf{x}_{k,j} \sim \mathcal{CN}(0, \mathbf{R}_{k,j}) \in \mathbb{C}^{NM\times 1}$, receiver thermal noise, $\mathbf{x}_{k,n} \sim \mathcal{CN}(0, \mathbf{R}_{k,n}) \in \mathbb{C}^{NM\times 1}$, and possible targets $\mathbf{x}_{k,s} = \sigma_s \mathbf{s}_{ts} \in \mathbb{C}^{NM\times 1}$, where σ_s is the intensity of the target and $\mathbf{s}_{ts}$ is a space-time steering vector. In this thesis, we assume a complex Gaussian distribution of the response of the clutter interference, the jamming interference and the thermal noise. This is the most common assumption in the literature [4]. However, algorithms using other distributions than the complex Gaussian has been presented. Common for such algorithms is to describe the clutter interference using more heavy-tailed distributions [6], [7].

In a detector, a test statistics is formed by multiplication of the radar snapshot in the cell-under-test with a weight vector. Thus, the test statistics is $y_k = |\mathbf{w}_k^H \mathbf{x}_k|$, where $\mathbf{w}_k$ is a weight vector and Hermitian transpose is denoted using the superscript 'H'. Detectors will further be introduced in Chapter 4. However, a useful measure of performance is to consider the signal-to-interference-and-noise-ratio (SINR) of the test statistics. The SINR is defined as

$$\mathrm{SINR} = \frac{|\mathbf{w}_k^H \mathbf{x}_{k,s}|^2}{E\{|\mathbf{w}_k^H(\mathbf{x}_{k,j} + \mathbf{x}_{k,c}) + \mathbf{w}_k^H \mathbf{x}_{k,n}|^2\}} \tag{2.4}$$

where $E\{\cdot\}$ denotes expected value [8]. The SINR has shown to have a one-to-one relationship with detector performance. Consequently, maximizing the SINR implies also maximizing the performance of a detector. Therefore, the SINR is seen as a useful measure to compare the performance of radar signal processing algorithms [9]–[13].

Clutter interference

Clutter interference is defined as a conglomerate of unwanted radar echos. Depending on the application, the term clutter includes echos from different objects. In the radar application considered in this thesis, echoes from the ground, trees, clouds, mountains and man-made buildings are considered to be clutter interference [4].

Jammer interference

Signals that are emitted by another system than the own radar, and interfere with the radar at the used frequency band, are considered to be jamming signals. Jamming can both be intentional, as an electronic warfare technique, and unintentional, as when a friendly radar emits a signal at the same frequency band as the own radar platform. Interference from jamming signals differs from clutter interference in the sense that jammers originates from platforms actively emitting signals [4].

A jamming signal is usually observed by the receiver via the direct signal. A direct signal only travels one way between the emitting platform and the receiving antenna. Radar echos will travel two ways; to the target and back to the radar. Thus, jamming signals can cause large problems as they can have a significantly larger power compared to received target echoes. Moreover, jamming signals usually originates from a distinct spatial direction and have a broad Doppler spectra [8].

CHAPTER 3

Spectral Estimation

In this chapter, spectral estimation is briefly introduced.

3.1 Basic theory

In many applications, the physical processes can be described by a weighted sum of the individual frequency components of the system. Such processes can be found in radio astronomy, magnetic resonance imaging (MRI), sonars and wireless communication applications [14]. For radars, the gridless angle-Doppler-range estimation problem has this formulation [15]. In line spectral estimation, the aim is to find the individual frequency components from measurements of such processes. Consequently, this implies to find estimates to multiple frequencies, and their corresponding amplitudes and possible damping factors.

Techniques of spectral estimation can be divided into two groups; *non-parametric* techniques and *parametric* techniques [14]. In a non-parametric technique the frequency spectrum of the signal is analyzed. It can be obtained via a discrete Fourier transform (DTF) or via the (windowed) periodogram. From the frequency spectrum, a non-parametric technique obtains the dominant frequency components contained in the analyzed signal. A parametric spectral estimation technique assumes that the signal can be well described by a model. The aim of such a technique is to fit the model towards the available measurements. The frequency components can then be derived from the model. Various parametric techniques for frequency estimation has been investigated in appended Paper A, and is briefly discussed in the subsequent section.

3.2 Parametric spectral estimation

Assume that a, noise free, signal y can be described by the time-discrete model:

$$y(k) = \sum_{i=1}^{n} \alpha_i e^{(j\omega_i + \beta_i)k} \tag{3.1}$$

where $\omega_i \in \mathbb{R}^n$, $\alpha_i \in \mathbb{C}^n$ and $\beta_i \in \mathbb{R}^n$ are unknown frequencies, amplitudes and damping factors, respectively. Consequently, from a sequence $\{y(k)\}_{k=1}^{K}$, the aim is to find ω_i, α_i and β_i for all $i \in [1, \ldots, n]$.

Several approaches on how to find the frequency components as well as their amplitudes and possible damping factors has been presented in the literature. Such approaches includes non-linear least squares minimization and maximum likelihood techniques [16]. Those techniques usually includes the formation of a cost function, and a non-linear optimization procedure which finds the optimal solution to the cost function. However, for problems of higher order, such frameworks may be hard to solve due to local minimums in the cost function, which may affect the accuracy of the estimate. Therefore, we will further present subspace methods which may be better suited to solve problems of higher order.

Introducing the matrix $A = \text{diag}[e^{(j\omega_1+\beta_1)k}, e^{(j\omega_2+\beta_2)k}, \ldots, e^{(j\omega_n+\beta_n)k}]$, and the vectors $C = [\alpha_1, \alpha_2, \ldots, \alpha_n]^T$ and $x_0 = [1, 1, \ldots, 1]^T$, where diag$[\cdot]$ represent a diagonal matrix. The signal (3.1) can be rewritten as

$$x(k+1) = Ax(k), \quad x(0) = x_0 \tag{3.2}$$

$$y(k) = Cx(k) \tag{3.3}$$

Then, let Y_0 be a matrix of Hankel structure with m rows that follows

$m > n$ and $K - m + 1 > n$ to the sequence $\{y(k)\}_{k=1}^{K}$. The matrix Y_0 can be factorized using the state-space representation as

$$Y_0 = \mathcal{O}\mathcal{C} \tag{3.4}$$

where $\mathcal{O} = [C, CA, CA^2, \ldots, CA^{m-1}]^T$ and $\mathcal{C} = [x_0, Ax_0, \ldots, A^{K-m}x_0]^T$. Consequently, forming a Hankel structure from the available sequence $\{y(k)\}_{k=1}^{K}$ and performing the factorization (3.4), the unknown parameters can be identified from the factors of Y_0.

In a practical application, the corresponding sequence $\{y(k)\}_{k=1}^{K}$ would be corrupted by measurement noise. This would imply that the exact factorization (3.4) is not possible [17]. To overcome this, several techniques has been presented which finds an approximation to Y_0 in the case of measurement noise. In the appended Paper A, three techniques which approximates Y_0 has been investigated. Those are ESPRIT [18], [19], a method which relaxes the rank constraint by a nuclear norm [20] and a method which both follows the rank constraint and imposing a Hankel structure on the estimate [21]. The main contribution of Paper A is an illustration of how the problem formulation and rank constraint management affect the accuracy of frequency component estimates.

CHAPTER 4

Detection

Target detection serve as the primary objective of a radar system. In this chapter, we briefly explain binary hypotheses testing, and introduce the matched filter and the Kelly detector.

4.1 Binary Hypothesis Testing

A detection problem arises from the situation when a measurement can originate from a number of possible events. The objective of the detection problem is to find which event that generated the measurement. If the measurement is seen as a realization of a stochastic variable, a rational framework to determine between the possible events is to perform hypothesis testing. In hypothesis testing, the measurement is used to form a test statistics which is evaluated against different decision boundaries. Depending on the relationship between the test statistics and the decision boundaries, the measurement is declared to originate from a specific hypothesis. If two possible events can generate the measurement, binary hypothesis testing can be performed.

Consider a measurement $\mathbf{x}$ that is a realization of two possible events. Denote the possible events as E_0 and E_1. To determine which event the measurement is an realization of, we formulate two hypotheses:

$$H_0 : \mathbf{x} \in E_0 \tag{4.1}$$

$$H_1 : \mathbf{x} \in E_1 \tag{4.2}$$

where H_0 is the *null-hypothesis* where the measurement $\mathbf{x}$ originates from event E_0, and H_1 is the *alternative hypothesis* where the measurement originates from event E_1. As $\mathbf{x}$ is seen as a realization of a stochastic variable, denote the corresponding probability density function in the case of the null-hypothesis as $p_{\mathbf{x}|H_0}(\mathbf{x})$ and in case of the alternative hypothesis as $p_{\mathbf{x}|H_1}(\mathbf{x})$.

The performance measures of a binary hypothesis test includes the probability of detection (P_D) and the probability of false alarm (P_{FA}). Those are given by

$$P_D = Pr[H_1 \text{ chosen}|H_1 \text{ is true}] = \int_{\mathbf{x}\in\Lambda(\mathbf{x})>\gamma} p_{\mathbf{x}|H_1}(\mathbf{x})d\mathbf{x} \tag{4.3}$$

$$P_{FA} = Pr[H_1 \text{ chosen}|H_0 \text{ is true}] = \int_{\mathbf{x}\in\Lambda(\mathbf{x})>\gamma} p_{\mathbf{x}|H_0}(\mathbf{x})d\mathbf{x} \tag{4.4}$$

where $\Lambda(\mathbf{x})$ defines the test statistics of the detector, and γ denotes the decision threshold between the hypotheses. Figure 4.1 includes a visualization of the two hypotheses, the decision threshold and corresponding P_D and P_{FA}. Note, as can be seen in Figure 4.1, a given decision threshold defines the tradeoff between maximizing the P_D and minimizing the P_{FA}.

To determine between the two hypotheses, i.e. to determine what the function $\Lambda(\mathbf{x})$ is, the Neyman-Pearson criteria (NPC) can be used [9], [16]. The objective of the NPC is to maximize the P_D while keeping $P_{FA} < \varepsilon$, where ε is the maximum false alarm rate the detector can tolerate. The optimal solution to the NPC is a decision mechanism known as the likelihood ratio test (LRT) [16]. It is given by

$$\Lambda^*(\mathbf{x}) = \frac{p_{\mathbf{x}|H_1}(\mathbf{x})}{p_{\mathbf{x}|H_0}(\mathbf{x})} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{4.5}$$

where the superscript '$*$' denotes optimality. Consequently, the LRT evaluates the ratio of the likelihood functions of the alternative hypothesis and the null-hypothesis with a threshold. If the test statistics exceeds the threshold, the

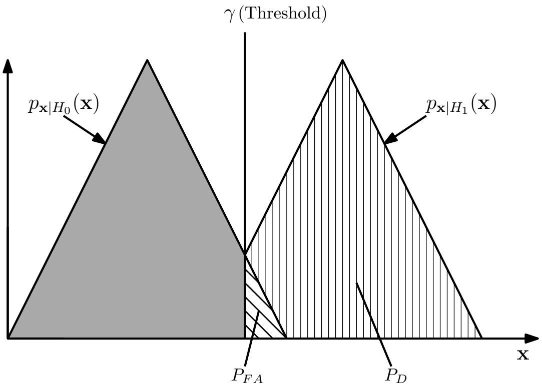

Figure 4.1: Illustration of binary hypotheses testing.

processor declares the alternative hypothesis, otherwise it declares the null-hypothesis.

To evaluate the LRT, the processor needs full knowledge of both $p_{\mathbf{x}|H_1}(\mathbf{x})$ and $p_{\mathbf{x}|H_0}(\mathbf{x})$. In a practical situation, it is unlikely that the processor holds that information. Rather, some parameters which defines the probability density functions are likely to be unknown. In such cases, the unknown parameter can be replaced by an estimate of the parameter. If the unknown parameter is replaced by its maximum likelihood estimate, and is used in the framework (4.5), a generalized likelihood ratio test (GLRT) has been obtained. Thus, the GLRT is given by

$$\Lambda(\mathbf{x}) = \frac{\max_{\theta \in \Omega_1} p_{\mathbf{x}|H_1}(\mathbf{x}|\theta_1)}{\max_{\theta \in \Omega_0} p_{\mathbf{x}|H_0}(\mathbf{x}|\theta_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{4.6}$$

where θ_0 and θ_1 are unknown parameter vectors within the sets Ω_0 and Ω_1, respectively. Note, (4.6) may not be an optimal solution to the NPC if Ω_0 and/or Ω_1 are finite sets [22].

4.2 Detectors used in Radar Systems

Now, consider the binary hypothesis test used in radar systems. Consequently, a radar system must determine the presence or the absence of a target in the currently investigated cell-under-test (CUT). If range bin k corresponds to the CUT, the binary hypotheses becomes

$$H_0 : \mathbf{x}_{k/H_0} = \mathbf{x}_{k,c} + \mathbf{x}_{k,j} + \mathbf{x}_{k,n} \tag{4.7}$$

$$H_1 : \mathbf{x}_{k/H_1} = \mathbf{x}_{k,s} + \mathbf{x}_{k,c} + \mathbf{x}_{k,j} + \mathbf{x}_{k,n} \tag{4.8}$$

Thus, under the null-hypothesis a target is absent in range bin k, while under the alternative hypothesis a target is present in range bin k. Under the assumption of complex Gaussian distributions of the clutter, the jammers and the thermal noise, as introduced in Chapter 2, the probability density functions for the null-hypothesis is $p_{\mathbf{x}|H_0}(\mathbf{x_k}) = \mathcal{CN}(0, \mathbf{R_k})$ and for the alternative hypothesis is $p_{\mathbf{x}|H_0}(\mathbf{x}_k) = \mathcal{CN}(\sigma_s \mathbf{s}_{ts}, \mathbf{R_k})$. For mutual statistically uncorrelated clutter interference, jamming interference and thermal noise, we have $\mathbf{R}_k = \mathbf{R}_{k,c} + \mathbf{R}_{k,j} + \mathbf{R}_{k,n}$.

The scalar output used in the detector as test statistic is $y_k = \mathbf{w}_k^H \mathbf{x}_k$. From a SINR perspective, the optimal weight vector which mitigates the influence of interference and thermal noise is $\mathbf{w}_k = \mu \mathbf{R}_k^{-1} \mathbf{s}_{ts}$, where $\mathbf{s}_{ts}$ is the space-time steering vector to the currently investigated cell-under-test [11]. The optimal weight vector $\mathbf{w}_k$ includes an arbitrary scalar μ. While a particular choice of μ does not affect the SINR, some are more beneficial in subsequent processing. A commonly used detector is

$$|\mathbf{w}_k^H \mathbf{x}_k|^2 = \frac{|\mathbf{s}_{ts}^H \mathbf{R}_k^{-1} \mathbf{x}_k|^2}{\mathbf{s}_{ts}^H \mathbf{R}_k^{-1} \mathbf{s}_{ts}} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{4.9}$$

where $\mu = 1/\sqrt{\mathbf{s}_{ts}^H \mathbf{R}_k^{-1} \mathbf{s}_{ts}}$. This detector is called the *matched filter*, and have two useful properties. First, it solves the LRT and is consequently the optimal detector. Secondly, it has the constant false alarm rate (CFAR) property [23]. It implies a normalization in the test statistics which compensates for the power of the interference and thermal noise to maintain a constant probability of false alarm.

Similarly, a GLRT derived by Kelly,

$$\frac{|\mathbf{s}_{ts}^H \hat{\mathbf{R}}_k^{-1} \mathbf{x}_k|^2}{\mathbf{s}_{ts}^H \hat{\mathbf{R}}_k^{-1} \mathbf{s}_{ts} (1 + \frac{1}{K} \mathbf{x}_k \hat{\mathbf{R}}_k^{-1} \mathbf{x}_k)} \underset{H_0}{\overset{H_1}{\gtrless}} K\gamma \tag{4.10}$$

where $\hat{\mathbf{R}}_k$ is an estimate of $\mathbf{R}_k$ and K is the number of radar snapshots used in the estimate $\hat{\mathbf{R}}_k$ [24]. Further discussions on estimates of $\mathbf{R}_k$ is presented in Chapter 5. The Kelly detector holds the CFAR property, and can be used when the space-time covariance matrix is not known.

CHAPTER 5

Space-Time Adaptive Processing

In this chapter we introduce the space-time adaptive processing (STAP) technique. It can be used to limit the effects of interference in airborne radar systems.

5.1 Basic Theory

For ground based radar systems, the echoes that originates from the ground, buildings and other non-moving objects will have a zero Doppler frequency. Thus, such clutter can trivially be mitigated by applying a notch filter to the zero Doppler frequency. However, for airborne radar systems the motion of the radar platform will introduce a relative velocity to the non-moving objects. Thus, echoes from clutter becomes angle-Doppler dependent, and may coincide with possible targets at a common Doppler frequency. In this chapter we present, space-time adaptive processing, a commonly used technique to mitigate the influence of interference in airborne radar systems [25].

A STAP-algorithm is a multidimensional filtering technique which combines signals from an array antenna with multiple pulses of coherent waveforms. The main objective of STAP is to find an estimate to the distribution of the interference. This to mitigate the interference while still preserving the strength of the desired signal. If the mitigation is properly performed, possible targets can be detected more easily [10], [11].

Recall from Chapter 4 that the test statistics used in a radar detector included a weight vector, and that the optimal weight vector from a SINR perspective is $\mathbf{w}_k = \mu \mathbf{R}_k^{-1} \mathbf{s}_{ts}$, where $\mathbf{R}_k$ is the space-time covariance matrix of the two hypotheses. In a practical situation, the covariance matrix $\mathbf{R}_k$ may not be known to the processor and consequently must be estimated. Several techniques of how the space-time covariance matrix is estimated has been presented in the literature. In the following sections, we introduce a few of the different estimation techniques.

5.2 Sample Covariance Matrix Estimate

Consider a set of radar observations $\bar{\mathbf{x}} = \{\mathbf{x}_k\}_{k=1}^{K}$ from K range bins in a neighborhood to the CUT. Assume all snapshots in $\bar{\mathbf{x}}$ are complex Gaussian and statistically independent and identically distributed (IID) with a zero mean. Thus, $\mathbf{x}_k \sim \mathcal{CN}(0, \mathbf{R})$ for all $k \in [1, K]$. The set $\bar{\mathbf{x}}$ represents observations obtained from a monostatic side-looking radar [13].

The maximum likelihood estimate of the space-time covariance matrix is obtained by solving the following optimization problem:

$$\hat{\mathbf{R}}_{\mathrm{ML}} = \arg\max_{\mathbf{R}} L(\bar{\mathbf{x}}|\mathbf{R}) \tag{5.1}$$

where $L(\bar{\mathbf{x}}|\mathbf{R})$ is the associated likelihood function to the distribution of $\bar{\mathbf{x}}$. The optimal solution to (5.1) is

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}_k \mathbf{x}_k^H \tag{5.2}$$

The solution (5.2) is known as the *sample covariance matrix* (SCM) estimate. The SCM has been shown to reach a −3 dB SINR loss compared to the SINR of the clairvoyant covariance matrix if at least $K = 2NM$ snapshots are used in the SCM estimate [10]. This is known as the Reed-Mallet-Brennan (RMB) rule.

For radar observations in real world applications, secondary data may not be

IID over the range dimension. Two factors mainly affects the response; clutter heterogeneity and geometry-induced range variations. The clutter heterogeneity arise from landscape variations, which is an effect of shadowing, clutter reflectivity and terrain discretes. In such cases, the processor can be aided with information about the current environment. This thesis does not cover heterogeneous clutter, but it is further explained in [26], [27]. The geometry-induced effect arises from the relative array configuration compared to platform heading. This introduces variations of the clutter in the angle-Doppler domain over range. Angle-Doppler variations occur for radar configurations other than the side-looking monostatic case. Thus, in forward-looking arrays [25], [28], circular arrays [29] and bistatic configurations [25], [30], [31]. As STAP-algorithms commonly base the space-time covariance estimate on secondary data collected from the range dimension, the angle-Doppler variations degrades the accuracy of such estimates.

Now, consider a new set of secondary data $\tilde{\mathbf{x}} = \{\mathbf{x}_k\}_{k=1}^{K}$ gathered from K range bins in a neighborhood from the CUT. The distribution of $\tilde{\mathbf{x}}$ follows $\mathbf{x}_k \sim \mathcal{CN}(0, \mathbf{R}_k)$ for all $k \in [1, K]$. Note specifically that the covariance matrix of $\tilde{\mathbf{x}}$ is range dependent. The expected value of a SCM applied to secondary data $\tilde{\mathbf{x}}$ becomes

$$E[\hat{\mathbf{R}}_{\text{CUT}}] = \frac{1}{K}\sum_{k=1}^{K} E[\mathbf{x}_k \mathbf{x}_k^H] = \frac{1}{K}\sum_{k=1}^{K} \mathbf{R}_k \tag{5.3}$$

Thus, the adaptive filter becomes the average behavior to the covariance matrices of the ingoing secondary data, rather than the best suited for the considered range bin.

Several algorithms has been presented which addresses the complications of range dependent secondary data. The algorithms can mainly be divided into three categories based on their processing technique. In the first category, we find algorithms that tries to limit the variations within the secondary data itself. That can be accomplished by only consider secondary data in a close vicinity to the CUT, followed by some dimension reduction technique [31], [32]. In the two other categories, the variations are included in the processing. Thus, in the second category, presumed variations in the secondary data is modeled. That includes a time-varying weight scheme where temporal variations in the secondary data is modeled [31], [33], [34]. In the last category, transformations are formed which aims to homogenize the distribution of snapshots over range. Such techniques are called Doppler warping. Several transformations of Doppler warping has been presented [35]–[41]. For instance, in a registration approach, the direction-Doppler (DD) curve of the snapshots is adjusted via curve fitting towards a reference DD-curve in an other range bin [39]. While in the Adaptive Angle-Doppler Compensation (A^2DC) method, the dominant subspace of each range bin is homogenized towards a common reference subspace. This is accomplished by rotating the eigenvector corresponding to the dominant eigenvalue in each range bin towards the corresponding dominant eigenvalue in the reference range bin [40], [41].

5.3 Reduced dimension techniques

In the previous sections, we describe processing techniques applied directly towards N spatial channels and M coherent pulses. This direct formulation is known as the joint-domain STAP. However, from an implementation perspective, the joint-domain STAP is of limited use. Two factors mainly limit the usability of joint-domain STAP; sample support and computational burden. As an example, consider the numerical parameters used in the public MCARM data collection program [42]. There, they have used $N = 22$ array channels and $M = 128$ coherent pulses. To fulfill the Reed Mallet Brennan (RMB)-rule, the necessary sample support need to be $2NM = 5632$ [10]. This heavily exceeds the available 630 snapshots in the MCARM dataset. Additionally, the SCM is associated with a computational complexity of $\mathcal{O}(N^3M^3)$. Consequently, using large number of array channels processing multiple pulses complicates implementations in radar system. To circumvent these limitations, sophisticated techniques which reduce the size of the necessary sample support and lower the computational burden can be utilized. Here, we will introduce one of these techniques; reduced-dimension techniques. However, one other commonly used technique is reduced-rank techniques, where the low rank nature of clutter and jammers is utilized. This is further presented in [43], [44]

In a Reduced-Dimension (RD) STAP, the radar observations are filtered with data-independent transformations before the STAP [13]. The objective is to reduce the number of adaptive degrees of freedom (DoF), which will reduce the necessary sample support and computational burden. Thus, for a space-time secondary data $\mathbf{x}_k$, the transformed data vector becomes

$$\breve{\mathbf{x}}_k = \mathbf{T}_k^H \mathbf{x}_k \tag{5.4}$$

where $\mathbf{T}_k \in C^{NM \times J}$ is a transformation matrix. The transformed data vector $\breve{\mathbf{x}}_k$ has dimensions $J \times 1$ where $J < NM$. Corresponding STAP algorithms, as described in Section 5.2, is then applied to $\breve{\mathbf{x}}_k$.

Multiple choices of $\mathbf{T}$ are possible. Naturally, the best choice provides an effective combination of DoFs to mitigate the interference while minimizing the computational burden, the required sample support and at the same time not reduce the power of the echo signal from the potential target in the CUT. Common selections of transformations includes traditional radar signal processing building blocks as Doppler processing and beamforming [45], [46].

CHAPTER 6

Summary of included papers

This chapter provides a summary of the included papers.

6.1 Paper A

Jacob Klintberg, Tomas McKelvey
An Improved Method for Parametric Spectral Estimation
Published in IEEE International Conference on Acoustics, Speech and Signal Processing,
vol. 17, no. 10, pp. 5551–5555, May. 2019.
©2019 IEEE DOI: 10.1109/ICASSP.2019.8683111 .

In this paper, the problem of parametric spectral estimation is considered. Three different methods are investigated for frequency component estimation from noisy measurements. The main contribution of the work is an visualization of how matrix rank and structure constraints affect the accuracy of the frequency estimates.

6.2 Paper B

Jacob Klintberg, Tomas McKelvey, Patrik Dammert
Mitigation of Ground Clutter in Airborne Bistatic Radar Systems
Published in IEEE Sensor Array and Multichannel Signal Processing Workshop,
pp. 1–5, June. 2020.
©2020 IEEE DOI: 10.1109/SAM48682.2020.9104314 .

Space-time adaptive processing algorithms are dependent on accurate estimates of the space-time covariance matrix to mitigate the influence of interference and thermal noise. In this paper, we investigate the sensitivity of a covariance matrix estimate which is based upon the current bistatic radar scenario. Thus, knowledge of the parameters defining the scenario can be used, via a model, to calculate the covariance matrix. However, in practical applications the processor may not have knowledge of the scenario parameters. Therefore, in this work we investigate the sensitivity of the space-time covariance matrix towards deviations in the scenario parameters. The sensitivity is measured via detector performance, and compared via numerical simulations with other state-of-the-art covariance matrix estimation methods.

6.3 Paper C

Jacob Klintberg, Tomas McKelvey, Patrik Dammert
A Parametric Approach to Space-Time Adaptive Processing in Bistatic Radar Systems
Submitted for publication in IEEE Transactions on Aerospace and Electronic Systems,
Dec. 2020.

This paper considers estimation of the space-time covariance matrix for airborne bistatic radar systems. As secondary data is range dependent for such systems, the estimation problem becomes somewhat involved. In this paper, we present a method which estimates the covariance matrix via the current radar scenario. As the parameters defining the scenario is unknown to the processor an maximum likelihood estimate of the scenario is obtained using all available secondary data. The covariance matrix is given via a model describing the scenario. If used in a detector, this approach would approximately represent a generalized likelihood ratio test as unknowns are replaced with their maximum likelihood estimates based on secondary data. Numerical simulations indicates significantly reduced SINR-losses with the presented method compared to other state-of-the-art methods.

CHAPTER 7

Concluding Remarks and Future Work

Target detection serve as one of the primary objectives of a radar system. To obtain the necessary information for target detections, signal processing algorithms are used to filter the electromagnetic echoes from the radar surroundings. In this thesis, two different signal processing algorithms are considered; spectral estimation and space-time adaptive processing.

The spectral estimation is considered for signals that can be represented by a parametric model. The objective is to estimate frequency components and their amplitudes and damping factors. In radar systems, the problem of gridless angle-Doppler-range estimation of the echoes has this formulation. In this thesis, we investigate three different methods for this problem. The difference between the methods are the conditions they impose of the frequency estimates. The main contribution of this work is a visualization on how management of matrix rank and structure affect the accuracy of the estimate.

For airborne radar systems, the behavior of the clutter interference will depend on the radar configuration. A side-looking monostatic radar will observe snapshots that are statistically independent and identically distributed for all ranges. This implies that a STAP-algorithm can obtain the maximum likelihood estimate of the space-time covariance matrix of the interference and the thermal noise. However, other radar configurations than the side-looking monostatic case, will not observe secondary snapshots that are identically distributed over the range dimension. A maximum likelihood estimate of the space-time covariance can consequently not be obtained without any additional processing of the radar snapshots.

In this thesis, we investigate and present a method which estimates the space-time covariance matrix based upon a model which describes the current bistatic radar scenario. The current radar scenario is defined by parameters connected to the transmitting platform, the receiving platform and to the reflectivity of the clutter interference. If the parameters are known, the space-time covariance matrix can be calculated via the model describing the scenario. However, the parameters defining the scenario are typically unknown to the processor in real applications. Therefore, we present a method which finds a maximum likelihood estimate to the scenario parameters using the available radar observations. If used in a detector, this approach approximately would represent a generalized likelihood ratio test as unknowns are replaced by their maximum likelihood estimates based on all secondary data. Numerical simulations indicates improved performance of the presented method compared to other state-of-the-art methods.

In the work regarding mitigation of interference in bistatic systems, several assumptions has been made upon the response of the clutter interference and knowledge of the radar models. Thus, we have assumed a homogeneous clutter response and no mismatch between algorithm model assumptions and the model generating the radar observations. These assumptions may not be valid in practical radar use cases. Therefore, the future research directions of this work aim to investigate deviations from these assumptions and the possible impacts it will have on the presented method. Consequently, the presented method will be evaluated on heterogeneous clutter observations, and a mismatch between the model assumed by the algorithm and the model used to generate the observations. The intention of such evaluation is to imitate the behavior of radar responses obtained from real scenarios. Ultimately, the evaluation will answer the question if the used assumptions are sufficient for real world applications, or if additional considerations has to be made on the algorithm for it to be a useful tool in practical applications.

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